Cut-elimination for $\mathbf{SLLM_m}$

Recall the Lambek Calculus with lax monoidal soft subexponentials, $\mathbf{SLLM}_m$ below:

$$A ::= A \in At \mid A \cdot A \mid A/A \mid A\backslash A \mid !A \mid \nabla A$$

$$\frac{}{A \longrightarrow A} \qquad \frac{\Gamma \longrightarrow A \quad \Sigma, A, \Delta \longrightarrow B}{\Sigma, \Gamma, \Delta \longrightarrow A}\ cut$$

$$\frac{\Gamma \longrightarrow A \quad \Sigma_1, B, \Sigma_2 \longrightarrow C}{\Sigma_1, \Gamma, A\backslash B, \Sigma_2 \longrightarrow C}\ \backslash_L \qquad \frac{A, \Gamma \longrightarrow B}{\Gamma \longrightarrow A\backslash B}\ \backslash_R$$

$$\frac{\Gamma \longrightarrow A \quad \Sigma_1, B, \Sigma_2 \longrightarrow C}{\Sigma_1, B/A, \Gamma, \Sigma_2 \longrightarrow C}\ /_L \qquad \frac{\Gamma, A \longrightarrow B}{\Gamma \longrightarrow B/A}\ /_R$$

$$\frac{\Gamma_1, A, B, \Gamma_2 \longrightarrow C}{\Gamma_1, A \cdot B, \Gamma_2 \longrightarrow C}\ \cdot_L \qquad \frac{\Gamma_1 \longrightarrow A \quad \Gamma_2 \longrightarrow B}{\Gamma_1, \Gamma_2 \longrightarrow A \cdot B}\ \cdot_R$$

$$\frac{\Gamma_1, \overbrace{A, A, \ldots, A}^{n \text{ times}}, \Gamma_2 \longrightarrow B}{\Gamma_1, !A, \Gamma_2 \longrightarrow B}\ \widetilde{!}_L \qquad \frac{A \longrightarrow B}{!A \longrightarrow !B}\ \widetilde{!}_R$$

$$\frac{\Gamma_1, A, \Gamma_2 \longrightarrow B}{\Gamma_1, \nabla A, \Gamma_2 \longrightarrow B}\ \nabla_L \qquad \frac{A \longrightarrow B}{\nabla A \longrightarrow \nabla B}\ \widetilde{\nabla}_R$$

$$\frac{\Gamma_1, \Gamma_2, \nabla A, \Gamma_3 \longrightarrow B}{\Gamma_1, \nabla A, \Gamma_2, \Gamma_3 \longrightarrow B}\ perm \qquad \frac{\Gamma_1, \nabla A, \Gamma_2, \Gamma_3 \longrightarrow B}{\Gamma_1, \Gamma_2, \nabla A, \Gamma_3 \longrightarrow B}\ perm'$$

Table 1: Sequent calculus presentation of $\mathbf{SLLM}$, where $1 \leq n \leq k_0$.

Theorem 1. *(cut) can be eliminated from $\mathbf{SLLM}_m$. That is, any proof in $\mathbf{SLLM}_m$ containing a cut can be converted into an equivalent proof without cuts.*

Proof. We perform a standard induction on the depths of the proofs of the sequents in the cut-rule. Throughout this proof, we will say that the left hand sequent $\Gamma \longrightarrow A$ has proof π, and the right-hand sequent $\Sigma, A, \Delta \longrightarrow B$ has proof τ. The final rule of a proof ω will be called $\mathrm{fin}(\omega)$.

Base case

If π and τ are simply instances of the axiom, that is, (**??**) becomes:

$$\dfrac{\overline{A \longrightarrow A} \qquad \overline{A \longrightarrow A}}{A \longrightarrow A}\ cut \tag{1}$$

we can simply transform (1) to $\overline{A \longrightarrow A}$. This completes the base-case of the induction.

Principal cuts

The following cases are those where the cut formula is principal in both $\mathrm{fin}(\pi)$ and $\mathrm{fin}(\tau)$.

1. $\mathrm{fin}(\pi) = \backslash_R$ and $\mathrm{fin}(\tau) = \backslash_L$. That is, this instance of cut is the following:

$$\dfrac{\dfrac{\dfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\ \backslash_R \qquad \dfrac{\dfrac{\tau'}{\Delta \longrightarrow A} \qquad \dfrac{\tau''}{\Sigma, B, \Theta \longrightarrow C}}{\Sigma, \Delta, A\backslash B, \Theta \longrightarrow C}\ \backslash_L}{\Sigma, \Delta, \Gamma, \Theta \longrightarrow C}\ cut \tag{2}$$

   We transform this proof to the following one containing two cuts where the cut formulas are of lower complexity:

$$\dfrac{\dfrac{\tau'}{\Delta \longrightarrow A} \qquad \dfrac{\dfrac{\pi'}{A, \Gamma \longrightarrow B} \qquad \dfrac{\tau''}{\Sigma, B, \Theta \longrightarrow C}}{\Sigma, A, \Gamma, \Theta \longrightarrow C}\ cut}{\Sigma, \Delta, \Gamma, \Theta \longrightarrow C}\ cut \tag{3}$$

2. $\mathrm{fin}(\pi) = \cdot_R$ and $\mathrm{fin}(\tau) = \cdot_L$. That is, this instance of cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \qquad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\ \cdot_R \qquad \dfrac{\dfrac{\tau'}{\Delta, A, B, \Theta \longrightarrow C}}{\Delta, A \cdot B, \Theta \longrightarrow C}\ \cdot_L}{\Delta, \Gamma, \Sigma, \Theta \longrightarrow C}\ cut \tag{4}$$

   which we transform to the following proof, containing two cuts of lower complexity:

$$\dfrac{\dfrac{\pi''}{\Sigma \longrightarrow B} \qquad \dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \qquad \dfrac{\tau'}{\Delta, A, B, \Theta \longrightarrow C}}{\Delta, \Gamma, B, \Theta \longrightarrow C}\ cut}{\Delta, \Gamma, \Sigma, \Theta \longrightarrow C}\ cut \tag{5}$$

3. $\mathrm{fin}(\pi) = !_R$ and $\mathrm{fin}(\tau) = !_L$ That is, this instance of cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\ !_R \qquad \dfrac{\dfrac{\tau'}{\Sigma, \Delta, A, \Theta, \Delta, A, \Theta, \ldots, \Delta, A, \Theta, \chi \longrightarrow B}}{\Sigma, !\Delta, !A, !\Theta, \chi \longrightarrow B}\ !_L}{\Sigma, !\Delta, !\Gamma, !\Theta, \chi \longrightarrow B}\ cut \tag{6}$$

which we transform into the following proof containing n cuts of lower complexity.

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \qquad \dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\tau'}{\Sigma, \Delta, A, \Theta, \Delta, A, \Theta, \ldots, \Delta, A, \Theta, \chi \longrightarrow B}}{\Sigma, \Delta, \Gamma, \Theta, \Delta, A, \Theta, \ldots, \Delta, A, \Theta, \chi \longrightarrow B}\ cut}{\vdots}\ cut}{}}{\Sigma, \Delta, \Gamma, \Theta, \Delta, \Gamma, \Theta, \ldots, \Delta, A, \Theta, \chi \longrightarrow B}\ cut}{\Sigma, \Delta, \Gamma, \Theta, \Delta, \Gamma, \Theta, \ldots, \Delta, \Gamma, \Theta, \chi \longrightarrow B}\ cut}{\Sigma, !\Delta, !\Gamma, !\Theta, \chi \longrightarrow B}\ !_L \tag{7}$$

4. $\mathrm{fin}(\pi) = \nabla_R$ and $\mathrm{fin}(\tau) = \nabla_L$ That is, this instance of cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\ \nabla_R \qquad \dfrac{\dfrac{\tau'}{\Sigma, A, \Delta \longrightarrow B}}{\Sigma, \nabla A, \Delta \longrightarrow B}\ \nabla_L}{\Sigma, \nabla\Gamma, \Delta \longrightarrow B}\ cut \tag{8}$$

which we transform to the following proof of lower complexity, assuming that $\Gamma = A_1, \ldots, A_m$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\tau'}{\Sigma, A, \Delta \longrightarrow B}}{\Sigma, \Gamma, \Delta \longrightarrow B}\ cut}{\Sigma, A_1, A_2, \ldots, A_m, \Delta \longrightarrow B}}{\Sigma, \nabla A_1, A_2, \ldots, A_m, \Delta \longrightarrow B}\ \nabla_L}{\vdots}\ \nabla_L}{\Sigma, \nabla A_1, \nabla A_2, \ldots, A_m, \Delta \longrightarrow B}\ \nabla_L}{\dfrac{\Sigma, \nabla A_1, \nabla A_2, \ldots, \nabla A_m, \Delta \longrightarrow B}{\Sigma, \nabla\Gamma, \Delta \longrightarrow B}}\ \nabla_L \tag{9}$$

5. $\mathrm{fin}(\pi) = \nabla_R$ and $\mathrm{fin}(\tau) = perm$ That is, this instance of cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\ \nabla_R \qquad \dfrac{\dfrac{\tau'}{\Sigma, \nabla A, \Delta, \Theta \longrightarrow B}}{\Sigma, \Delta, \nabla A, \Theta \longrightarrow B}\ perm}{\Sigma, \Delta, \nabla\Gamma, \Theta \longrightarrow B}\ cut \tag{10}$$

which we transform to the following proof which moves the cut up by one step, assuming that $\Gamma = A_1, \ldots, A_m$.

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\nabla_R \qquad \cfrac{\tau'}{\Sigma, \nabla A, \Delta, \Theta \longrightarrow B}}{\Sigma, \nabla\Gamma, \Delta, \Theta \longrightarrow B}\mathit{cut}}{\Sigma, \nabla A_1, \nabla A_2, \ldots, \nabla A_m, \Delta, \Theta \longrightarrow B}}{\vdots}\mathit{perm}}{\Sigma, \nabla A_1, \nabla A_2, \Delta, \ldots, \nabla A_m, \Theta \longrightarrow B}\mathit{perm}}{\Sigma, \nabla A_1, \Delta, \nabla A_2, \ldots, \nabla A_m, \Theta \longrightarrow B}\mathit{perm}}{\Sigma, \Delta, \nabla A_1, \nabla A_2, \ldots, \nabla A_m, \Theta \longrightarrow B}\mathit{perm}}{\Sigma, \Delta, \nabla\Gamma, \Theta \longrightarrow B} \tag{11}$$

Non-principal cuts

We split the non-principal cuts into three cases: Case I - where π ends in a right hand rule, Case II - where π ends in a left hand rule, and Case III, where pi ends in $perm$ or $perm'$. For each case we will analyse all possible final rules of τ. Since there are 12 rules of the calculus, excluding the axiom, there are $12 \cdot 12 = 144$ non-principal cuts. However in our exposition we will only consider $\backslash$ and $perm$, since the treatments of / and $perm'$ are symmetric, reducing the following exposition to $9 \cdot 9 = 81$ cases.

Case I

1. $\text{fin}(\pi) = \backslash_R$, which makes the cut look like

$$\cfrac{\cfrac{\cfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_R \qquad \cfrac{\tau}{\Sigma[A\backslash B] \longrightarrow C}}{\Sigma[\Gamma] \longrightarrow C}\mathit{cut} \tag{12}$$

We now follow by analysing the possible final rules of τ.

i. $\text{fin}(\tau) = \backslash_L$

$$\cfrac{\cfrac{\cfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_R \qquad \cfrac{\cfrac{\tau'}{\Sigma \longrightarrow C} \qquad \cfrac{\tau''}{\Delta, D, \Theta \longrightarrow E}}{\Delta, \Sigma, C\backslash D, \Theta \longrightarrow E}\backslash_L}{\Delta, \Sigma, C\backslash D, \Theta \longrightarrow E}\mathit{cut} \tag{13}$$

where $A\backslash B$ is contained in either Δ, Σ or Θ. This is transformed to the proof where $A\backslash B$ is cut with Δ, Σ or Θ prior to the application of the $\backslash_L$-rule.

a. $A\backslash B$ contained in Σ; that is $\Sigma = \Sigma[A\backslash B]$ we transform to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_R \quad \dfrac{\tau'}{\Sigma[A\backslash B] \longrightarrow C}}{\Sigma[\Gamma] \longrightarrow C}\,cut \quad \dfrac{\tau''}{\Delta, D, \Theta \longrightarrow E}}{\Delta, \Sigma[\Gamma], C\backslash D, \Theta \longrightarrow E}\backslash_L \tag{14}$$

b. $A\backslash B$ contained in Δ; that is $\Delta = \Delta[A\backslash B]$ we transform to

$$\dfrac{\dfrac{\tau'}{\Sigma \longrightarrow C} \quad \dfrac{\dfrac{\dfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_R \quad \dfrac{\tau''}{\Delta[A\backslash B], D, \Theta \longrightarrow E}}{\Delta[\Gamma], D, \Theta \longrightarrow E}\,cut}{\Delta[\Gamma], \Sigma, C\backslash D, \Theta \longrightarrow E}\backslash_L \tag{15}$$

c. $A\backslash B$ contained in Θ; that is $\Theta = \Theta[A\backslash B]$ we transform to

$$\dfrac{\dfrac{\tau'}{\Sigma \longrightarrow C} \quad \dfrac{\dfrac{\dfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_R \quad \dfrac{\tau'}{\Delta, D, \Theta[A\backslash B] \longrightarrow E}}{\Delta, D, \Theta[\Gamma] \longrightarrow E}\,cut}{\Delta, \Sigma, C\backslash D, \Theta[\Gamma] \longrightarrow E}\backslash_L \tag{16}$$

ii. $\mathrm{fin}(\tau) = \backslash_R$ that is, the cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_R \quad \dfrac{\dfrac{\tau'}{C, \Sigma[A\backslash B] \longrightarrow D}}{\Sigma[A\backslash B] \longrightarrow C\backslash D}\backslash_R}{\Sigma[\Gamma] \longrightarrow C\backslash D}\,cut \tag{17}$$

which is transformed into

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_R \quad \dfrac{\tau'}{C, \Sigma[A\backslash B] \longrightarrow D}}{C, \Sigma[\Gamma] \longrightarrow D}\,cut}{\Sigma[\Gamma] \longrightarrow C\backslash D}\backslash_R \tag{18}$$

iii. $\mathrm{fin}(\tau) = \cdot_L$ whose cut is:

$$\dfrac{\dfrac{\dfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_R \quad \dfrac{\dfrac{\tau'}{\Sigma, C, D, \Delta \longrightarrow E}}{\Sigma, C \cdot D, \Delta \longrightarrow E}\cdot_L}{\Sigma, C \cdot D, \Delta \longrightarrow E}\,cut \tag{19}$$

where $A\backslash B$ is either contained in Σ or Δ, giving us the two cases:

a. $\Sigma = \Sigma[A\backslash B]$. This we transform to:

$$\cfrac{\cfrac{\cfrac{\pi'}{A,\Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_L \qquad \cfrac{\tau'}{\Sigma[A\backslash B], C, D, \Delta \longrightarrow E}}{\cfrac{\Sigma[\Gamma], C, D, \Delta \longrightarrow E}{\Sigma[\Gamma], C \cdot D, \Delta \longrightarrow E}\cdot_L}\, cut \tag{20}$$

b. $\Delta = \Delta[A\backslash B]$. This we transform to:

$$\cfrac{\cfrac{\cfrac{\pi'}{A,\Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_L \qquad \cfrac{\tau'}{\Sigma, C, D, \Delta[A\backslash B] \longrightarrow E}}{\cfrac{\Sigma, C, D, \Delta[\Gamma] \longrightarrow E}{\Sigma, C \cdot D, \Delta[\Gamma] \longrightarrow E}\cdot_L}\, cut \tag{21}$$

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iv. $\mathrm{fin}(\tau) = \cdot_R$. In this case the cut is of the form

$$\cfrac{\cfrac{\cfrac{\pi'}{A,\Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_R \qquad \cfrac{\tau'}{\Sigma \longrightarrow C} \qquad \cfrac{\tau''}{\Delta \longrightarrow D}}{\Sigma, \Delta \longrightarrow C \cdot D}\, cut \tag{22}$$

where $A\backslash B$ is contained in either Σ or Δ, giving us the following two cases:

a. $\Sigma = \Sigma[A\backslash B]$. This we transform to

$$\cfrac{\cfrac{\cfrac{\cfrac{\pi'}{A,\Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_R \qquad \cfrac{\tau'}{\Sigma[A\backslash B] \longrightarrow C}}{\Sigma[\Gamma] \longrightarrow C}\, cut \qquad \cfrac{\tau''}{\Delta \longrightarrow D}}{\Sigma[\Gamma], \Delta \longrightarrow C \cdot D}\cdot_R \tag{23}$$

b. $\Delta = \Delta[A\backslash B]$. This we transform to

$$\cfrac{\cfrac{\tau'}{\Sigma \longrightarrow C} \qquad \cfrac{\cfrac{\cfrac{\pi'}{A,\Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_R \qquad \cfrac{\tau''}{\Delta[A\backslash B] \longrightarrow D}}{\Delta[\Gamma] \longrightarrow D}\, cut}{\Sigma, \Delta[\Gamma] \longrightarrow C \cdot D}\cdot_R \tag{24}$$

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v. $\mathrm{fin}(\tau) = !_L$. In this case the cut is of the form

$$\cfrac{\cfrac{\cfrac{\pi'}{A,\Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\backslash_R \qquad \cfrac{\cfrac{\tau'}{\Sigma, \Delta, \Delta, \ldots, \Delta, \Theta \longrightarrow C}}{\Sigma, !\Delta, \Theta \longrightarrow C}\, !_L}{\Sigma, !\Delta, \Theta \longrightarrow C}\, cut \tag{25}$$

where $A\backslash B$ is contained in Σ or Θ, giving us the following two cases:

a. $\Sigma = \Sigma[A\backslash B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\pi'}{A,\Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\,\backslash_R \qquad \dfrac{\tau'}{\Sigma[A\backslash B],\Delta,\Delta,\ldots,\Delta,\Theta \longrightarrow C}}{\dfrac{\Sigma[\Gamma],\Delta,\Delta,\ldots,\Delta,\Theta \longrightarrow C}{\Sigma[\Gamma],!\Delta,\Theta \longrightarrow C}\,!_L}\,cut \tag{26}$$

b. $\Theta = \Theta[A\backslash B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\pi'}{A,\Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\,\backslash_R \qquad \dfrac{\tau'}{\Sigma,\Delta,\Delta,\ldots,\Delta,\Theta[A\backslash B] \longrightarrow C}}{\dfrac{\Sigma,\Delta,\Delta,\ldots,\Delta,\Theta[\Gamma] \longrightarrow C}{\Sigma,!\Delta,\Theta[\Gamma] \longrightarrow C}\,!_L}\,cut \tag{27}$$

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vi. $\mathrm{fin}(\tau) = !_R$. In this case no non-principal cut is possible.

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vii. $\mathrm{fin}(\tau) = \nabla_L$. In this case the cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{A,\Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\,\backslash_R \qquad \dfrac{\dfrac{\tau'}{\Sigma,C,\Delta \longrightarrow D}}{\Sigma,\nabla C,\Delta \longrightarrow D}\,\nabla_L}{\Sigma,\nabla C,\Delta \longrightarrow D}\,cut \tag{28}$$

where either Σ or Δ contain $A\backslash B$, giving us the following two cases:

i. $\Sigma = \Sigma[A\backslash B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\pi'}{A,\Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\,\backslash_R \qquad \dfrac{\tau'}{\Sigma[A\backslash B],C,\Delta \longrightarrow D}}{\dfrac{\Sigma[\Gamma],C,\Delta \longrightarrow D}{\Sigma[\Gamma],\nabla C,\Delta \longrightarrow D}\,\nabla_L}\,cut \tag{29}$$

ii. $\Delta = \Delta[A\backslash B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\pi'}{A,\Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\,\backslash_R \qquad \dfrac{\tau'}{\Sigma,C,\Delta[A\backslash B] \longrightarrow D}}{\dfrac{\Sigma,C,\Delta[\Gamma] \longrightarrow D}{\Sigma,\nabla C,\Delta[\Gamma] \longrightarrow D}\,\nabla_L}\,cut \tag{30}$$

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viii. $\mathrm{fin}(\tau) = \nabla_R$. In this case no cut is possible.

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ix. $\mathrm{fin}(\tau) = perm$. In this case the cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\,\backslash_R \qquad \dfrac{\dfrac{\tau'}{\Sigma, \Delta, \nabla C, \Theta \longrightarrow D}}{\Sigma, \nabla C, \Delta, \Theta \longrightarrow D}\,perm}{\Sigma, \Delta, \nabla C, \Theta \longrightarrow D}\,cut \tag{31}$$

where $A\backslash B$ is contained in either Σ, Δ or Θ, giving us the following three cases.

a. $\Sigma = \Sigma[A\backslash B]$. This we transform into

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\,\backslash_R \qquad \dfrac{\tau'}{\Sigma[A\backslash B], \nabla C, \Delta, \Theta \longrightarrow D}}{\Sigma[\Gamma], \nabla C, \Delta, \Theta \longrightarrow D}\,cut}{\Sigma[\Gamma], \Delta, \nabla C, \Theta \longrightarrow D}\,perm \tag{32}$$

b. $\Delta = \Delta[A\backslash B]$. This we transform into

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\,\backslash_R \qquad \dfrac{\tau'}{\Sigma, \nabla C, \Delta[A\backslash B], \Theta \longrightarrow D}}{\Sigma, \nabla C, \Delta[\Gamma], \Theta \longrightarrow D}\,cut}{\Sigma, \Delta[\Gamma], \nabla C, \Theta \longrightarrow D}\,perm \tag{33}$$

c. $\Theta = \Theta[A\backslash B]$. This we transform into

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{A, \Gamma \longrightarrow B}}{\Gamma \longrightarrow A\backslash B}\,\backslash_R \qquad \dfrac{\tau'}{\Sigma, \nabla C, \Delta, \Theta[A\backslash B] \longrightarrow D}}{\Sigma, \nabla C, \Delta, \Theta[\Gamma] \longrightarrow D}\,cut}{\Sigma, \Delta, \nabla C, \Theta[\Gamma] \longrightarrow D}\,perm \tag{34}$$

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2. $\mathrm{fin}(\pi) = \cdot_R$ In this case the cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\,\cdot_R \qquad \dfrac{\tau}{\Delta[A \cdot B] \longrightarrow C}}{\Delta[\Gamma, \Sigma] \longrightarrow C}\,cut \tag{35}$$

We now analyse the possible final rules of τ.

i. $\mathrm{fin}(\tau) = \backslash_L$. In this case the cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\cdot_R \quad \dfrac{\dfrac{\tau'}{\Delta \longrightarrow C} \quad \dfrac{\tau''}{\Theta, D, \chi \longrightarrow E}}{\Theta, \Delta, C\backslash D, \chi \longrightarrow E}\backslash_L}{\Theta, \Delta, C\backslash D, \chi \longrightarrow E}\,cut \tag{36}$$

where either Θ, Δ or χ contain $A \cdot B$, giving us the following three cases.

a. $\Theta = \Theta[A \cdot B]$. This we transform into

$$\dfrac{\dfrac{\tau'}{\Delta \longrightarrow C} \quad \dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\cdot_R \quad \dfrac{\tau''}{\Theta[A \cdot B], D, \chi \longrightarrow E}}{\Theta[\Gamma, \Sigma], D, \chi \longrightarrow E}\,cut}{\Theta[\Gamma, \Sigma], \Delta, C\backslash D, \chi \longrightarrow E}\backslash_L \tag{37}$$

b. $\Delta = \Delta[A \cdot B]$. This we transform into

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\cdot_R \quad \dfrac{\tau'}{\Delta[A \cdot B] \longrightarrow C}}{\Delta[\Gamma, \Sigma] \longrightarrow C}\,cut \quad \dfrac{\tau''}{\Theta, D, \chi \longrightarrow E}}{\Theta, \Delta[\Gamma, \Sigma], C\backslash D, \chi \longrightarrow E}\backslash_L \tag{38}$$

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ii. $\mathrm{fin}(\tau) = \backslash_R$. This cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\cdot_R \quad \dfrac{\dfrac{\tau'}{C, \Delta[A \cdot B] \longrightarrow D}}{\Delta[A \cdot B] \longrightarrow C\backslash D}\backslash_R}{\Delta[\Gamma, \Sigma] \longrightarrow C\backslash D}\,cut \tag{39}$$

which we transform to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\cdot_R \quad \dfrac{\tau'}{C, \Delta[A \cdot B] \longrightarrow D}}{C, \Delta[\Gamma, \Sigma] \longrightarrow D}\,cut}{\Delta[\Gamma, \Sigma] \longrightarrow C\backslash D}\backslash_R \tag{40}$$

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iii. $\mathrm{fin}(\tau) = \cdot_L$. This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\cdot_R \quad \dfrac{\dfrac{\tau'}{\Delta, C, D, \Theta \longrightarrow E}}{\Delta, C \cdot D, \Theta \longrightarrow E}\cdot_L}{\Delta, C \cdot D, \Theta \longrightarrow E}\,cut \tag{41}$$

where either Δ or Θ contain $A \cdot B$, giving us the following two cases

a. $\Delta = \Delta[A \cdot B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\,\cdot_R \quad \dfrac{\tau'}{\Delta[A \cdot B], C, D, \Theta \longrightarrow E}}{\Delta[\Gamma, \Sigma], C, D, \Theta \longrightarrow E}\,cut}{\Delta[\Gamma, \Sigma], C \cdot D, \Theta \longrightarrow E}\,\cdot_L \tag{42}$$

b. $\Theta = \Theta[A \cdot B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\,\cdot_R \quad \dfrac{\tau'}{\Delta, C, D, \Theta[A \cdot B] \longrightarrow E}}{\Delta, C, D, \Theta[\Gamma, \Sigma] \longrightarrow E}\,cut}{\Delta, C \cdot D, \Theta[\Gamma, \Sigma] \longrightarrow E}\,\cdot_L \tag{43}$$

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iv. $\mathrm{fin}(\tau) = \cdot_R$. This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\,\cdot_R \quad \dfrac{\dfrac{\tau'}{\Delta \longrightarrow C} \quad \dfrac{\tau''}{\Theta \longrightarrow D}}{\Delta, \Theta \longrightarrow C \cdot D}\,\cdot_R}{\Delta, \Theta \longrightarrow C \cdot D}\,cut \tag{44}$$

where either Δ or Θ contain the formula $A \cdot B$, giving us the following two cases.

a. $\Delta = \Delta[A \cdot B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\,\cdot_R \quad \dfrac{\tau'}{\Delta[A \cdot B] \longrightarrow C}}{\Delta[\Gamma, \Sigma] \longrightarrow C}\,cut \quad \dfrac{\tau''}{\Theta \longrightarrow D}}{\Delta[\Gamma, \Sigma], \Theta \longrightarrow C \cdot D}\,\cdot_R \tag{45}$$

b. $\Theta = \Theta[A \cdot B]$. This we transform to

$$\dfrac{\dfrac{\tau'}{\Delta \longrightarrow C} \quad \dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\,\cdot_R \quad \dfrac{\tau''}{\Theta[A \cdot B] \longrightarrow D}}{\Theta[\Gamma, \Sigma] \longrightarrow D}\,cut}{\Delta, \Theta[\Gamma, \Sigma] \longrightarrow C \cdot D}\,\cdot_R \tag{46}$$

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v. $\mathrm{fin}(\tau) = !_L$. This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\,\cdot_R \quad \dfrac{\dfrac{\tau'}{\Delta, \Theta, \Theta, \ldots, \Theta, \chi \longrightarrow C}}{\Delta, !\Theta, \chi \longrightarrow C}\,!_L}{\Delta, !\Theta, \chi \longrightarrow C}\,cut \tag{47}$$

where $A \cdot B$ is contained in either Δ or χ, giving us the following two cases

a. $\Delta = \Delta[A \cdot B]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\ \cdot_R \quad \dfrac{\tau'}{\Delta[A \cdot B], \Theta, \Theta, \ldots, \Theta, \chi \longrightarrow C}}{\dfrac{\Delta[\Gamma, \Sigma], \Theta, \Theta, \ldots, \Theta, \chi \longrightarrow C}{\Delta[\Gamma, \Sigma], !\Theta, \chi \longrightarrow C}\ !_L}\ cut \tag{48}$$

b. $\chi = \chi[A \cdot B]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\ \cdot_R \quad \dfrac{\tau'}{\Delta, \Theta, \Theta, \ldots, \Theta, \chi[A \cdot B] \longrightarrow C}}{\dfrac{\Delta, \Theta, \Theta, \ldots, \Theta, \chi[\Gamma, \Sigma] \longrightarrow C}{\Delta, !\Theta, \chi[\Gamma, \Sigma] \longrightarrow C}\ !_L}\ cut \tag{49}$$

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vi. $\mathrm{fin}(\tau) = !_R$. In this case no cut is possible.

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vii. $\mathrm{fin}(\tau) = \nabla_L$. This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\ \cdot_R \quad \dfrac{\dfrac{\tau'}{\Delta, C, \Theta \longrightarrow D}}{\Delta, \nabla C, \Theta \longrightarrow D}\ \nabla_L}{\Delta, \nabla C, \Theta \longrightarrow D}\ cut \tag{50}$$

where either Δ or Θ contain the formula $A \cdot B$, giving us the following two cases

a. $\Delta = \Delta[A \cdot B]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\ \cdot_R \quad \dfrac{\tau'}{\Delta[A \cdot B], C, \Theta \longrightarrow D}}{\dfrac{\Delta[\Gamma, \Sigma], C, \Theta \longrightarrow D}{\Delta[\Gamma, \Sigma], \nabla C, \Theta \longrightarrow D}\ \nabla_L}\ cut \tag{51}$$

b. $\Theta = \Theta[A \cdot B]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\ \cdot_R \quad \dfrac{\tau'}{\Delta, C, \Theta[A \cdot B] \longrightarrow D}}{\dfrac{\Delta, C, \Theta[\Gamma, \Sigma] \longrightarrow D}{\Delta, \nabla C, \Theta[\Gamma, \Sigma] \longrightarrow D}\ \nabla_L}\ cut \tag{52}$$

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viii. $\mathrm{fin}(\tau) = \nabla_R$ In this case no cut is possible.

---

ix. $\mathrm{fin}(\tau) = perm$. This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\,\cdot_R \quad \dfrac{\dfrac{\tau'}{\Delta, \Theta, \nabla C, \chi \longrightarrow D}}{\Delta, \nabla C, \Theta, \chi \longrightarrow D}\,\mathit{perm}}{\Delta, \nabla C, \Theta, \chi \longrightarrow D}\,\mathit{cut} \tag{53}$$

where either Δ, Θ or χ contain the formula $A \cdot B$, giving us the following three cases.

i. $\Delta = \Delta[A \cdot B]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A} \quad \dfrac{\pi''}{\Sigma \longrightarrow B}}{\Gamma, \Sigma \longrightarrow A \cdot B}\,\cdot_R \quad \dfrac{\tau'}{\Delta[A \cdot B], \Theta, \nabla C, \chi \longrightarrow D}}{\Delta[\Gamma, \Sigma], \Theta, \nabla C, \chi \longrightarrow D}\,\mathit{cut}}{\Delta[\Gamma, \Sigma], \nabla C, \Theta, \chi \longrightarrow D}\,\mathit{perm} \tag{54}$$

3. $\mathrm{fin}(\pi) = !_R$. In this case the cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\tau}{\Delta[!A] \longrightarrow B}}{\Delta[!\Gamma] \longrightarrow B}\,\mathit{cut} \tag{55}$$

We now analyse the possible final rules of τ.

i. $\mathrm{fin}(\tau) = \backslash_L$. This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_L \quad \dfrac{\dfrac{\tau'}{\Sigma \longrightarrow B} \quad \dfrac{\tau''}{\Theta, C, \chi \longrightarrow D}}{\Theta, \Sigma, B\backslash C, \chi \longrightarrow D}\,\backslash_L}{\Theta, \Sigma, B\backslash C, \chi \longrightarrow D}\,\mathit{cut} \tag{56}$$

where Θ, Σ or χ contain the cut formula $!A$, giving us the following three cases.

i. $\Theta = \Theta[!A]$. This is transformed to

$$\dfrac{\dfrac{\tau'}{\Sigma \longrightarrow B} \quad \dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\tau''}{\Theta[!A], C, \chi \longrightarrow D}}{\Theta[!\Gamma], \Sigma, B\backslash C, \chi \longrightarrow D}\,\mathit{cut}}{\Theta[!\Gamma], \Sigma, B\backslash C, \chi \longrightarrow D}\,\backslash_L \tag{57}$$

ii. $\Sigma = \Sigma[!A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\tau'}{\Sigma[!A] \longrightarrow B}}{\Sigma[!\Gamma] \longrightarrow B}\,\mathit{cut} \quad \dfrac{\tau''}{\Theta, C, \chi \longrightarrow D}}{\Theta, \Sigma[!\Gamma], B\backslash C, \chi \longrightarrow D}\,\backslash_L \tag{58}$$

iii. $\chi = \chi[!A]$. This is transformed to

$$\dfrac{\dfrac{\tau'}{\Sigma \longrightarrow B} \quad \dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\tau''}{\Theta, C, \chi[!A] \longrightarrow D}}{\Theta, \Sigma, B\backslash C, \chi[!\Gamma] \longrightarrow D}\,cut}{\Theta, \Sigma, B\backslash C, \chi[!\Gamma] \longrightarrow D}\,\backslash_L \tag{59}$$

---

ii. $\mathrm{fin}(\tau) = \backslash_R$. This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\dfrac{\tau'}{B, \Sigma[!A] \longrightarrow C}}{\Sigma[!A] \longrightarrow B\backslash C}\,\backslash_R}{\Sigma[!\Gamma] \longrightarrow B\backslash C}\,cut \tag{60}$$

which is transformed into

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\tau'}{B, \Sigma[!A] \longrightarrow C}}{B, \Sigma[!\Gamma] \longrightarrow C}\,cut}{\Sigma[!\Gamma] \longrightarrow B\backslash C}\,\backslash_L \tag{61}$$

---

iii. $\mathrm{fin}(\tau) = \cdot_L$. In this case the cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\dfrac{\tau'}{\Sigma, B, C, \Delta \longrightarrow D}}{\Sigma, B \cdot C, \Delta \longrightarrow D}\,\cdot_L}{\Sigma, B \cdot C, \Delta \longrightarrow D}\,cut \tag{62}$$

where $!A$ is contained in either Σ or Δ, giving us the following two cases

i. $\Sigma = \Sigma[!A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\tau'}{\Sigma[!A], B, C, \Delta \longrightarrow D}}{\Sigma[!\Gamma], B, C, \Delta \longrightarrow D}\,cut}{\Sigma[!\Gamma], B \cdot C, \Delta \longrightarrow D}\,\cdot_L \tag{63}$$

ii. $\Delta = \Delta[!A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\tau'}{\Sigma, B, C, \Delta[!A] \longrightarrow D}}{\Sigma, B, C, \Delta[!\Gamma] \longrightarrow D}\,cut}{\Sigma, B \cdot C, \Delta[!\Gamma] \longrightarrow D}\,\cdot_L \tag{64}$$

iv. $\text{fin}(\tau) = \cdot_R$. This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\;!_R \quad \dfrac{\dfrac{\tau'}{\Sigma \longrightarrow B} \quad \dfrac{\tau''}{\Delta \longrightarrow C}}{\Sigma, \Delta \longrightarrow B \cdot C}\;\cdot_R}{\Sigma, \Delta \longrightarrow B \cdot C}\;cut \tag{65}$$

where the cut formula $!A$ is contained in Σ or Δ, giving us the following two cases

i. $\Sigma = \Sigma[!A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\;!_R \quad \dfrac{\tau'}{\Sigma[!A] \longrightarrow B}}{\Sigma[!\Gamma] \longrightarrow B}\;cut \quad \dfrac{\tau''}{\Delta \longrightarrow C}}{\Sigma[!\Gamma], \Delta \longrightarrow B \cdot C}\;\cdot_R \tag{66}$$

ii. $\Delta = \Delta[!A]$. This is transformed to

$$\dfrac{\dfrac{\tau'}{\Sigma \longrightarrow B}\;cut \quad \dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\;!_R \quad \dfrac{\tau''}{\Delta[!A] \longrightarrow C}}{\Delta[!\Gamma] \longrightarrow C}\;cut}{\Sigma, \Delta[!\Gamma] \longrightarrow B \cdot C}\;\cdot_R \tag{67}$$

---

v. $\text{fin}(\tau) = !_L$. This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\;!_R \quad \dfrac{\dfrac{\tau'}{\Sigma, \Delta, \Delta, \ldots, \Delta, \Theta \longrightarrow B}}{\Sigma, !\Delta, \Theta \longrightarrow B}\;!_L}{\Sigma, !\Delta, \Theta \longrightarrow B}\;cut \tag{68}$$

where the cut formula $!A$ is contained in either Σ or Θ, giving us the following two cases

a. $\Sigma = \Sigma[!A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\;!_R \quad \dfrac{\tau'}{\Sigma[!A], \Delta, \Delta, \ldots, \Delta, \Theta \longrightarrow B}}{\Sigma[!\Gamma], \Delta, \Delta, \ldots, \Delta, \Theta \longrightarrow B}\;cut}{\Sigma[!\Gamma], !\Delta, \Theta \longrightarrow B}\;!_L \tag{69}$$

b. $\Delta = \Delta[!A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\tau'}{\Sigma, \Delta, \Delta, \ldots, \Delta[!A], \Theta \longrightarrow B}}{\dfrac{\Sigma, \Delta, \Delta, \ldots, \Delta, \Theta[!\Gamma] \longrightarrow B}{\Sigma, !\Delta, \Theta[!\Gamma] \longrightarrow B}\,!_L}\,cut \tag{70}$$

---

vi. $\mathrm{fin}(\tau) = !_R$. In this case, no non-principal cut is possible.

---

vii. $\mathrm{fin}(\tau) = \nabla_L$ This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\dfrac{\tau'}{\Sigma, B, \Delta \longrightarrow C}}{\Sigma, \nabla B, \Delta \longrightarrow C}\,\nabla_L}{\Sigma, \nabla B, \Delta \longrightarrow C}\,cut \tag{71}$$

where the cut formula $!A$ is contained in either Σ or Δ, giving us the following two cases

a. $\Sigma = \Sigma[!A]$. This we transform into

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\tau'}{\Sigma[!A], B, \Delta \longrightarrow C}}{\dfrac{\Sigma[!\Gamma], B, \Delta \longrightarrow C}{\Sigma[!\Gamma], \nabla B, \Delta \longrightarrow C}\,\nabla_L}\,cut \tag{72}$$

b. $\Delta = \Delta[!A]$. This we transform into

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\tau'}{\Sigma, B, \Delta[!A] \longrightarrow C}}{\dfrac{\Sigma, B, \Delta[!\Gamma] \longrightarrow C}{\Sigma, \nabla B, \Delta[!\Gamma] \longrightarrow C}\,\nabla_L}\,cut \tag{73}$$

---

viii. $\mathrm{fin}(\tau) = \nabla_R$. In this case, no cut is possible.

---

ix. $\mathrm{fin}(\tau) = perm$ This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\,!_R \quad \dfrac{\dfrac{\tau'}{\Sigma, \Delta, \nabla B, \Theta \longrightarrow C}}{\Sigma, \nabla B, \Delta, \Theta \longrightarrow C}\,perm}{\Sigma, \nabla B, \Delta, \Theta \longrightarrow C}\,cut \tag{74}$$

where the cut formula $!A$ is contained in either Σ, Δ or Θ, giving us the following three cases

a. $\Sigma = \Sigma[!A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\ !_R \quad \dfrac{\tau'}{\Sigma[!A], \nabla B, \Delta, \Theta \longrightarrow C}}{\dfrac{\Sigma[!\Gamma], \Delta, \nabla B, \Theta \longrightarrow C}{\Sigma[!\Gamma], \nabla B, \Delta, \Theta \longrightarrow C}\ perm}\ cut \tag{75}$$

b. $\Delta = \Delta[!A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\ !_R \quad \dfrac{\tau'}{\Sigma, \nabla B, \Delta[!A], \Theta \longrightarrow C}}{\dfrac{\Sigma, \Delta[!\Gamma], \nabla B, \Theta \longrightarrow C}{\Sigma, \nabla B, \Delta[!\Gamma], \Theta \longrightarrow C}\ perm}\ cut \tag{76}$$

c. $\Theta = \Theta[!A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{!\Gamma \longrightarrow !A}\ !_R \quad \dfrac{\tau'}{\Sigma, \nabla B, \Delta, \Theta[!A] \longrightarrow C}}{\dfrac{\Sigma, \Delta, \nabla B, \Theta[!\Gamma] \longrightarrow C}{\Sigma, \nabla B, \Delta, \Theta[!\Gamma] \longrightarrow C}\ perm}\ cut \tag{77}$$

4. $\operatorname{fin}(\pi) = \nabla_R$. In this case the cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\ \nabla_R \quad \dfrac{\tau}{\Sigma[\nabla A] \longrightarrow B}}{\Sigma[\nabla\Gamma] \longrightarrow B}\ cut \tag{78}$$

i. $\operatorname{fin}(\tau) = \backslash_L$ This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\ \nabla_R \quad \dfrac{\dfrac{\tau'}{\Sigma \longrightarrow B} \quad \dfrac{\tau''}{\Delta, C, \Theta \longrightarrow D}}{\Delta, \Sigma, B\backslash C, \Theta \longrightarrow D}\ \backslash_L}{\Delta, \Sigma, B\backslash C, \Theta \longrightarrow D}\ cut \tag{79}$$

where the cut formula ∇A is contained in either Δ, Σ or Θ, giving us the three following cases.

i. $\Delta = \Delta[\nabla A]$. This is transformed to

$$\dfrac{\dfrac{\tau'}{\Sigma \longrightarrow B} \quad \dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\ \nabla_R \quad \dfrac{\tau''}{\Delta[\nabla A], C, \Theta \longrightarrow D}}{\Delta[\nabla\Gamma], C, \Theta \longrightarrow D}\ cut}{\Delta[\nabla\Gamma], \Sigma, B\backslash C, \Theta \longrightarrow D}\ \backslash_L \tag{80}$$

ii. $\Sigma = \Sigma[\nabla A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\nabla_R \quad \dfrac{\tau'}{\Sigma[\nabla A] \longrightarrow B}}{\Sigma[\nabla\Gamma] \longrightarrow B}\mathit{cut} \quad \dfrac{\tau''}{\Delta, C, \Theta \longrightarrow D}}{\Delta, \Sigma[\nabla\Gamma], B\backslash C, \Theta \longrightarrow D}\backslash_L \tag{81}$$

iii. $\Theta = \Theta[\nabla A]$. This is transformed to

$$\dfrac{\dfrac{\tau'}{\Sigma \longrightarrow B} \quad \dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\nabla_R \quad \dfrac{\tau''}{\Delta, C, \Theta[\nabla A] \longrightarrow D}}{\Delta, C, \Theta[\nabla\Gamma] \longrightarrow D}\mathit{cut}}{\Delta, \Sigma, B\backslash C, \Theta[\nabla\Gamma] \longrightarrow D}\backslash_L \tag{82}$$

---

ii. $\mathrm{fin}(\tau) = \backslash_R$. This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\nabla_R \quad \dfrac{\dfrac{\tau'}{B, \Sigma[\nabla A] \longrightarrow C}}{\Sigma[\nabla A] \longrightarrow B\backslash C}\backslash_R}{\Sigma[!\Gamma] \longrightarrow B\backslash C}\mathit{cut} \tag{83}$$

which is transformed into

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow !A}\nabla_R \quad \dfrac{\tau'}{B, \Sigma[!A] \longrightarrow C}}{B, \Sigma[\nabla\Gamma] \longrightarrow C}\mathit{cut}}{\Sigma[\nabla\Gamma] \longrightarrow B\backslash C}\backslash_L \tag{84}$$

---

iii. $\mathrm{fin}(\tau) = \cdot_L$. In this case the cut is

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\nabla_R \quad \dfrac{\dfrac{\tau'}{\Sigma, B, C, \Delta \longrightarrow D}}{\Sigma, B \cdot C, \Delta \longrightarrow D}\cdot_L}{\Sigma, B \cdot C, \Delta \longrightarrow D}\mathit{cut} \tag{85}$$

where $!A$ is contained in either Σ or Δ, giving us the following two cases

i. $\Sigma = \Sigma[\nabla A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow !A}\nabla_R \quad \dfrac{\tau'}{\Sigma[\nabla A], B, C, \Delta \longrightarrow D}}{\Sigma[\nabla\Gamma], B, C, \Delta \longrightarrow D}\mathit{cut}}{\Sigma[\nabla\Gamma], B \cdot C, \Delta \longrightarrow D}\cdot_L \tag{86}$$

ii. $\Delta = \Delta[!A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\tau'}{\Sigma, B, C, \Delta[\nabla A] \longrightarrow D}}{\dfrac{\Sigma, B, C, \Delta[\nabla\Gamma] \longrightarrow D}{\Sigma, B \cdot C, \Delta[\nabla\Gamma] \longrightarrow D}\,\cdot_L}\,cut \tag{87}$$

---

iv. $\mathrm{fin}(\tau) = \cdot_R$. This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\dfrac{\tau'}{\Sigma \longrightarrow B} \quad \dfrac{\tau''}{\Delta \longrightarrow C}}{\Sigma, \Delta \longrightarrow B \cdot C}\,\cdot_R}{\Sigma, \Delta \longrightarrow B \cdot C}\,cut \tag{88}$$

where the cut formula $!A$ is contained in Σ or Δ, giving us the following two cases

i. $\Sigma = \Sigma[\nabla A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\tau'}{\Sigma[\nabla A] \longrightarrow B}}{\Sigma[\nabla\Gamma] \longrightarrow B}\,cut \qquad \dfrac{\tau''}{\Delta \longrightarrow C}}{\Sigma[\nabla\Gamma], \Delta \longrightarrow B \cdot C}\,\cdot_R \tag{89}$$

ii. $\Delta = \Delta[!A]$. This is transformed to

$$\dfrac{\dfrac{\tau'}{\Sigma \longrightarrow B} \qquad \dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\tau''}{\Delta[\nabla A] \longrightarrow C}}{\Delta[\nabla\Gamma] \longrightarrow C}\,cut}{\Sigma, \Delta[\nabla\Gamma] \longrightarrow B \cdot C}\,\cdot_R \tag{90}$$

---

v. $\mathrm{fin}(\tau) = !_L$. This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\dfrac{\tau'}{\Sigma, \Delta, \Delta, \ldots, \Delta, \Theta \longrightarrow B}}{\Sigma, !\Delta, \Theta \longrightarrow B}\,!_L}{\Sigma, !\Delta, \Theta \longrightarrow B}\,cut \tag{91}$$

where the cut formula ∇A is contained in either Σ or Θ, giving us the following two cases

a. $\Sigma = \Sigma[!A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\tau'}{\Sigma[\nabla A], \Delta, \Delta, \ldots, \Delta, \Theta \longrightarrow B}}{\Sigma[\nabla\Gamma], \Delta, \Delta, \ldots, \Delta, \Theta \longrightarrow B}\,cut}{\Sigma[\nabla\Gamma], !\Delta, \Theta \longrightarrow B}\,!_L \tag{92}$$

b. $\Delta = \Delta[!A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\tau'}{\Sigma, \Delta, \Delta, \ldots, \Delta[\nabla A], \Theta \longrightarrow B}}{\dfrac{\Sigma, \Delta, \Delta, \ldots, \Delta, \Theta[\nabla\Gamma] \longrightarrow B}{\Sigma, !\Delta, \Theta[\nabla\Gamma] \longrightarrow B}\,!_L}\,cut \tag{93}$$

---

vi. $\mathrm{fin}(\tau) = !_R$. In this case no cut is possible.

---

vii. $\mathrm{fin}(\tau) = \nabla_L$ This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\dfrac{\tau'}{\Sigma, B, \Delta \longrightarrow C}}{\Sigma, \nabla B, \Delta \longrightarrow C}\,\nabla_L}{\Sigma, \nabla B, \Delta \longrightarrow C}\,cut \tag{94}$$

where the cut formula ∇A is contained in either Σ or Δ, giving us the following two cases

a. $\Sigma = \Sigma[!A]$. This we transform into

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\tau'}{\Sigma[\nabla A], B, \Delta \longrightarrow C}}{\dfrac{\Sigma[\nabla\Gamma], B, \Delta \longrightarrow C}{\Sigma[\nabla\Gamma], \nabla B, \Delta \longrightarrow C}\,\nabla_L}\,cut \tag{95}$$

b. $\Delta = \Delta[!A]$. This we transform into

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\tau'}{\Sigma, B, \Delta[\nabla A] \longrightarrow C}}{\dfrac{\Sigma, B, \Delta[\nabla\Gamma] \longrightarrow C}{\Sigma, \nabla B, \Delta[\nabla\Gamma] \longrightarrow C}\,\nabla_L}\,cut \tag{96}$$

---

viii. $\mathrm{fin}(\tau) = \nabla_R$. In this case no non-principal is possible.

---

ix. $\mathrm{fin}(\tau) = perm$ This cut looks like

$$\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\dfrac{\tau'}{\Sigma, \Delta, \nabla B, \Theta \longrightarrow C}}{\Sigma, \nabla B, \Delta, \Theta \longrightarrow C}\,perm}{\Sigma, \nabla B, \Delta, \Theta \longrightarrow C}\,cut \tag{97}$$

where the cut formula ∇A is contained in either Σ, Δ or Θ, giving us the following three cases

a. $\Sigma = \Sigma[\nabla A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\tau'}{\Sigma[\nabla A], \nabla B, \Delta, \Theta \longrightarrow C}}{\Sigma[\nabla\Gamma], \Delta, \nabla B, \Theta \longrightarrow C}\,\mathit{cut}}{\Sigma[\nabla\Gamma], \nabla B, \Delta, \Theta \longrightarrow C}\,\mathit{perm} \tag{98}$$

b. $\Delta = \Delta[\nabla A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\tau'}{\Sigma, \nabla B, \Delta[\nabla A], \Theta \longrightarrow C}}{\Sigma, \Delta[\nabla\Gamma], \nabla B, \Theta \longrightarrow C}\,\mathit{cut}}{\Sigma, \nabla B, \Delta[\nabla\Gamma], \Theta \longrightarrow C}\,\mathit{perm} \tag{99}$$

c. $\Theta = \Theta[!A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\dfrac{\pi'}{\Gamma \longrightarrow A}}{\nabla\Gamma \longrightarrow \nabla A}\,\nabla_R \qquad \dfrac{\tau'}{\Sigma, \nabla B, \Delta, \Theta[\nabla A] \longrightarrow C}}{\Sigma, \Delta, \nabla B, \Theta[\nabla\Gamma] \longrightarrow C}\,\mathit{cut}}{\Sigma, \nabla B, \Delta, \Theta[\nabla\Gamma] \longrightarrow C}\,\mathit{perm} \tag{100}$$

Case II

1. $\mathrm{fin}(\pi) = \backslash_L$. In this case the cut is

$$\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\,\backslash_L \qquad \Theta[C] \longrightarrow D}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow D}\,\mathit{cut} \tag{101}$$

We continue by analysing the possible final rules of τ.

i. $\mathrm{fin}(\tau) = \backslash_L$. This cut looks like

$$\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\,\backslash_L \qquad \dfrac{\Theta \longrightarrow D \quad \chi, E, \Psi \longrightarrow F}{\chi, \Theta, D\backslash E, \Psi \longrightarrow F}\,\backslash_L}{\chi, \Theta, D\backslash E, \Psi \longrightarrow F}\,\mathit{cut} \tag{102}$$

where the cut-formula C is contained in either χ, Θ or Ψ, giving us the following three cases:

a. $\chi = \chi[C]$. This we transform to

$$\dfrac{\Theta \longrightarrow D \qquad \dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \qquad \chi[C], E, \Psi \longrightarrow F}{\chi[\Sigma, \Gamma, A\backslash B, \Delta], E, \Psi \longrightarrow F}\, cut}{\chi[\Sigma, \Gamma, A\backslash B, \Delta], \Theta, D\backslash E, \Psi \longrightarrow F}\backslash_L \tag{103}$$

b. $\Theta = \Theta[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \qquad \Theta[C] \longrightarrow D}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow D}\, cut \qquad \chi, E, \Psi \longrightarrow F}{\chi, \Theta[\Sigma, \Gamma, A\backslash B, \Delta], D\backslash E, \Psi \longrightarrow F}\backslash_L \tag{104}$$

c. $\Psi = \Psi[C]$. This we transform to

$$\dfrac{\Theta \longrightarrow D \qquad \dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \qquad \chi, E, \Psi[C] \longrightarrow F}{\chi, E, \Psi[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow F}\, cut}{\chi, \Theta, D\backslash E, \Psi[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow F}\backslash_L \tag{105}$$

---

ii. $\mathrm{fin}(\tau) = \backslash_R$. In this case the cut is

$$\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \qquad \dfrac{D, \Theta[C] \longrightarrow E}{\Theta[C] \longrightarrow D\backslash E}\backslash_R}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow D\backslash E}\, cut \tag{106}$$

which is transformed to

$$\dfrac{\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \qquad D, \Theta[C] \longrightarrow E}{D, \Theta[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow E}\, cut}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow D\backslash E}\backslash_R \tag{107}$$

---

iii. $\mathrm{fin}(\tau) = \cdot_L$. In this case the cut is

$$\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \qquad \dfrac{\Theta, D \cdot E, \chi \longrightarrow F}{\Theta, D \cdot E, \chi \longrightarrow F}\cdot_L}{\Theta, D \cdot E, \chi \longrightarrow F}\, cut \tag{108}$$

where the cut formula C is contained in either Θ or χ, giving us the following two cases.

a. $\Theta = \Theta[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \quad \Theta[C], D, E, \chi \longrightarrow F}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta], D, E, \chi \longrightarrow F}\mathit{cut}}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta], D \cdot E, \chi \longrightarrow F}\cdot_L \tag{109}$$

b. $\chi = \chi[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \quad \Theta, D, E, \chi[C] \longrightarrow F}{\Theta, D, E, \chi[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow F}\mathit{cut}}{\Theta, D \cdot E, \chi[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow F}\cdot_L \tag{110}$$

---

iv. $\mathrm{fin}(\tau) = \cdot_R$. In this case the cut looks like

$$\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \quad \dfrac{\Theta \longrightarrow D \quad \chi \longrightarrow F}{\Theta, \chi \longrightarrow D \cdot F}\cdot_R}{\Theta, \chi \longrightarrow D \cdot F}\mathit{cut} \tag{111}$$

where the cut formula, C is contained in either Θ or χ, giving us the following two cases.

a. $\Theta = \Theta[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \quad \Theta[C] \longrightarrow D}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow D}\mathit{cut} \quad \chi \longrightarrow F}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta], \chi \longrightarrow D \cdot F}\cdot_R \tag{112}$$

b. $\chi = \chi[C]$. This we transform to

$$\dfrac{\Theta \longrightarrow D \quad \dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \quad \chi[C] \longrightarrow F}{\chi[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow F}\mathit{cut}}{\Theta, \chi[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow D \cdot F}\cdot_R \tag{113}$$

---

v. $\mathrm{fin}(\tau) = !_L$. In this case the cut looks like

$$\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \quad \dfrac{\Theta, \chi, \chi, \ldots, \chi, \Psi \longrightarrow D}{\Theta, !\chi, \Psi \longrightarrow D}!_L}{\Theta, !\chi, \Psi \longrightarrow D}\mathit{cut} \tag{114}$$

where the cut formula C is contained in either Θ or Ψ, giving us the following two cases.

a. $\Theta = \Theta[C]$. This we transform to

$$\cfrac{\cfrac{\cfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C} \backslash_L \quad \Theta[C], \chi, \chi, \ldots, \chi, \Psi \longrightarrow D}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta], \chi, \chi, \ldots, \chi, \Psi \longrightarrow D} cut}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta], !\chi, \Psi \longrightarrow D} !_L \tag{115}$$

b. $\Psi = \Psi[C]$. This we transform to

$$\cfrac{\cfrac{\cfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C} \backslash_L \quad \Theta, \chi, \chi, \ldots, \chi, \Psi[C] \longrightarrow D}{\Theta, \chi, \chi, \ldots, \chi, \Psi[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow D} cut}{\Theta, !\chi, \Psi[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow D} !_L \tag{116}$$

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vi. $\text{fin}(\tau) = !_R$. In this case no cut is possible

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vii. $\text{fin}(\tau) = \nabla_L$. In this case the cut looks like

$$\cfrac{\cfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C} \backslash_L \quad \cfrac{\Theta, D, \chi \longrightarrow E}{\Theta, \nabla D, \chi \longrightarrow E} \nabla_L}{\Theta, \nabla D, \chi \longrightarrow E} cut \tag{117}$$

where the cut formula C is contained in either Θ or Ψ, giving us the following two cases.

a. $\Theta = \Theta[C]$. This we transform to

$$\cfrac{\cfrac{\cfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C} \backslash_L \quad \Theta[C], D, \Psi \longrightarrow E}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta], D, \chi \longrightarrow E} cut}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta], \nabla D, \chi \longrightarrow E} \nabla_L \tag{118}$$

b. $\chi = \chi[C]$. This we transform to

$$\cfrac{\cfrac{\cfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C} \backslash_L \quad \Theta, D, \Psi[C] \longrightarrow E}{\Theta, D, \chi[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow E} cut}{\Theta, \nabla D, \chi[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow E} \nabla_L \tag{119}$$

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viii. $\text{fin}(\tau) = \nabla_R$. In this case no cut is possible.

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ix. $\text{fin}(\tau) = perm$. In this case the cut looks like

$$\cfrac{\cfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C} \backslash_L \quad \cfrac{\Theta, \chi, \nabla D, \Psi \longrightarrow E}{\Theta, \nabla D, \chi, \Psi \longrightarrow E} perm}{\Theta, \nabla D, \chi, \Psi \longrightarrow E} cut \tag{120}$$

where the cut formula C is contained in either Θ, χ or Ψ, giving us the following three cases.

a. $\Theta = \Theta[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \quad \Theta[C], \chi, \nabla D, \Psi \longrightarrow E}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta], \chi, \nabla D, \Psi \longrightarrow E}\, cut}{\Theta[\Sigma, \Gamma, A\backslash B, \Delta], \nabla D, \chi, \Psi \longrightarrow E}\, perm \tag{121}$$

b. $\chi = \chi[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \quad \Theta, \chi[C], \nabla D, \Psi \longrightarrow E}{\Theta, \chi[\Sigma, \Gamma, A\backslash B, \Delta], \nabla D, \Psi \longrightarrow E}\, cut}{\Theta, \nabla D, \chi[\Sigma, \Gamma, A\backslash B, \Delta], \Psi \longrightarrow E}\, perm \tag{122}$$

c. $\Psi = \Psi[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma \longrightarrow A \quad \Sigma, B, \Delta \longrightarrow C}{\Sigma, \Gamma, A\backslash B, \Delta \longrightarrow C}\backslash_L \quad \Theta, \chi, \nabla D, \Psi[C] \longrightarrow E}{\Theta, \chi, \nabla D, \Psi[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow E}\, cut}{\Theta, \nabla D, \chi, \Psi[\Sigma, \Gamma, A\backslash B, \Delta] \longrightarrow E}\, perm \tag{123}$$

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2. $\mathrm{fin}(\pi) = \cdot_L$ In this case the cut is

$$\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\cdot_L \Theta[C] \longrightarrow D}{\Theta[\Gamma, A \cdot B, \Sigma] \longrightarrow D}\, cut \tag{124}$$

We continue by analysing the possible final rules of τ.

i. $\mathrm{fin}(\tau) = \backslash_L$. In this case the cut is

$$\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\cdot_L \quad \dfrac{\chi \longrightarrow D \quad \Theta, E, \Psi \longrightarrow F}{\Theta, \chi, D\backslash E, \Psi \longrightarrow F}\backslash_L}{\Theta, \chi, D\backslash E, \Psi \longrightarrow F}\, cut \tag{125}$$

where the cut formula C is contained in either Θ, χ or Ψ, giving us the following three cases.

a. $\Theta = \Theta[C]$. This we transform to

$$\dfrac{\chi \longrightarrow D \qquad \dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\,\cdot_L \qquad \Theta[C], E, \Psi \longrightarrow F}{\Theta[\Gamma, A \cdot B, \Sigma], E, \Psi \longrightarrow F}\,cut}{\Theta[\Gamma, A \cdot B, \Sigma], \chi, D\backslash E, \Psi \longrightarrow F}\,\backslash_L \tag{126}$$

b. $\chi = \chi[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\,\cdot_L \qquad \chi[C] \longrightarrow D}{\chi[\Gamma, A \cdot B, \Sigma] \longrightarrow D}\,cut \qquad \Theta, E, \Psi \longrightarrow F}{\Theta, \chi[\Gamma, A \cdot B, \Sigma], D\backslash E, \Psi \longrightarrow F}\,\backslash_L \tag{127}$$

c. $\Psi = \Psi[C]$. This we transform to

$$\dfrac{\chi \longrightarrow D \qquad \dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\,\cdot_L \qquad \Theta, E, \Psi[C] \longrightarrow F}{\Theta, E, \Psi[\Gamma, A \cdot B, \Sigma] \longrightarrow F}\,cut}{\Theta, \chi, D\backslash E, \Psi[\Gamma, A \cdot B, \Sigma] \longrightarrow F}\,\backslash_L \tag{128}$$

---

ii. $\operatorname{fin}(\tau) = \backslash_R$. In this case the cut looks like

$$\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\,\cdot_L \qquad \dfrac{D, \Delta[C] \longrightarrow E}{\Delta[C] \longrightarrow D\backslash E}\,\backslash_R}{\Delta[\Gamma, A \cdot B, \Sigma] \longrightarrow D\backslash E}\,cut \tag{129}$$

which we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\,\cdot_L \qquad D, \Delta[C] \longrightarrow E}{D, \Delta[\Gamma, A \cdot B, \Sigma] \longrightarrow E}\,cut}{\Delta[\Gamma, A \cdot B, \Sigma] \longrightarrow D\backslash E}\,\backslash_R \tag{130}$$

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iii. $\operatorname{fin}(\tau) = \cdot_L$. This cut looks like

$$\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\,\cdot_L \qquad \dfrac{\Delta, D, E, \Theta \longrightarrow F}{\Delta, D \cdot E, \Theta \longrightarrow F}\,\cdot_L}{\Delta, D \cdot E, \Theta \longrightarrow F}\,cut \tag{131}$$

where the cut formula C is contained in either Δ or Θ, giving us the following two cases

a. $\Delta = \Delta[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\,\cdot_L \quad \Delta[C], D, E, \Theta \longrightarrow F}{\Delta[\Gamma, A \cdot B, \Sigma], D, E, \Theta \longrightarrow F}\,cut}{\Delta[\Gamma, A \cdot B, \Sigma], D \cdot E, \Theta \longrightarrow F}\,\cdot_L \tag{132}$$

b. $\Theta = \Theta[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\,\cdot_L \quad \Delta, D, E, \Theta[C] \longrightarrow F}{\Delta, D, E, \Theta[\Gamma, A \cdot B, \Sigma] \longrightarrow F}\,cut}{\Delta, D \cdot E, \Theta[\Gamma, A \cdot B, \Sigma] \longrightarrow F}\,\cdot_L \tag{133}$$

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iv. $\mathrm{fin}(\tau) = \cdot_R$. In this case the cut looks like

$$\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\,\cdot_L \quad \dfrac{\Delta \longrightarrow D \quad \Theta \longrightarrow E}{\Delta, \Theta \longrightarrow D \cdot E}\,\cdot_R}{\Delta, \Theta \longrightarrow D \cdot E}\,cut \tag{134}$$

where the cut formula C is contained in either Δ or Θ, giving us the following two cases

a. $\Delta = \Delta[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\,\cdot_L \quad \Delta[C] \longrightarrow D}{\Delta[\Gamma, A \cdot B, \Sigma] \longrightarrow D}\,cut \quad \Theta \longrightarrow E}{\Delta[\Gamma, A \cdot B, \Sigma], \Theta \longrightarrow D \cdot E}\,\cdot_R \tag{135}$$

b. $\Theta = \Theta[C]$. This we transform to

$$\dfrac{\Delta \longrightarrow D \quad \dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\,\cdot_L \quad \Theta[C] \longrightarrow D}{\Theta[\Gamma, A \cdot B, \Sigma] \longrightarrow E}\,cut}{\Delta, \Theta[\Gamma, A \cdot B, \Sigma] \longrightarrow D \cdot E}\,\cdot_R \tag{136}$$

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v. $\mathrm{fin}(\tau) = !_L$. In this case the cut looks like

$$\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\,\cdot_L \quad \dfrac{\Delta, \Theta, \Theta, \ldots, \Theta, \chi \longrightarrow D}{\Delta, !\Theta, \chi \longrightarrow D}\,!_L}{\Delta, !\Theta, \chi \longrightarrow D}\,cut \tag{137}$$

where the cut formula C is contained in either Δ or χ, giving us the following two cases

a. $\Delta = \Delta[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma} \cdot_L \quad \Delta[C], \Theta, \Theta, \ldots, \Theta, \chi \longrightarrow D}{\Delta[\Gamma, A \cdot B, \Sigma], \Theta, \Theta, \ldots, \Theta, \chi \longrightarrow D}\ cut}{\Delta[\Gamma, A \cdot B, \Sigma], !\Theta, \chi \longrightarrow D}\ !_L \tag{138}$$

b. $\chi = \chi[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma} \cdot_L \quad \Delta, \Theta, \Theta, \ldots, \Theta, \chi[C] \longrightarrow D}{\Delta, \Theta, \Theta, \ldots, \Theta, \chi[\Gamma, A \cdot B, \Sigma] \longrightarrow D}\ cut}{\Delta, !\Theta, \chi[\Gamma, A \cdot B, \Sigma] \longrightarrow D}\ !_L \tag{139}$$

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vi. $\mathrm{fin}(\tau) = !_R$. In this case no cut is possible.

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vii. $\mathrm{fin}(\tau) = \nabla_L$. In this case the cut looks like

$$\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C} \cdot_L \quad \dfrac{\Delta, D, \Theta \longrightarrow E}{\Delta, \nabla D, \Theta \longrightarrow E}\ \nabla_L}{\Delta, \nabla D, \Theta \longrightarrow E}\ cut \tag{140}$$

where the cut formula C is contained in either Δ or Θ giving us the following two cases.

a. $\Delta = \Delta[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C} \cdot_L \quad \Delta[C], D, \Theta \longrightarrow E}{\Delta[\Gamma, A \cdot B, \Sigma], D, \Theta \longrightarrow E}\ cut}{\Delta[\Gamma, A \cdot B, \Sigma], \nabla D, \Theta \longrightarrow E}\ \nabla_L \tag{141}$$

b. $\Theta = \Theta[C]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C} \cdot_L \quad \Delta, D, \Theta[C] \longrightarrow E}{\Delta, D, \Theta[\Gamma, A \cdot B, \Sigma] \longrightarrow E}\ cut}{\Delta, \nabla D, \Theta[\Gamma, A \cdot B, \Sigma] \longrightarrow E}\ \nabla_L \tag{142}$$

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viii. $\mathrm{fin}(\tau) = \nabla_R$. In this case no cut is possible.

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ix. $\mathrm{fin}(\tau) = perm$. In this case the cut looks like

$$\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C} \cdot_L \quad \dfrac{\Delta, \Theta, \nabla D, \chi \longrightarrow E}{\Delta, \nabla D, \Theta, \chi \longrightarrow E}\ perm}{\Delta, \nabla D, \Theta, \chi \longrightarrow E}\ cut \tag{143}$$

where the cut formula is contained in Δ, Θ or χ, giving us the following three cases

a. $\Delta = \Delta[C]$. This we transform to

$$\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\ \cdot_L \qquad \Delta[C], \nabla D, \Theta, \chi \longrightarrow E}{\dfrac{\Delta[\Gamma, A \cdot B, \Sigma], \Theta, \nabla D, \chi \longrightarrow E}{\Delta[\Gamma, A \cdot B, \Sigma], \nabla D, \Theta, \chi \longrightarrow E}\ perm}\ cut \tag{144}$$

b. $\Theta = \Theta[C]$. This we transform to

$$\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\ \cdot_L \qquad \Delta, \nabla D, \Theta[C], \chi \longrightarrow E}{\dfrac{\Delta, \Theta[\Gamma, A \cdot B, \Sigma], \nabla D, \chi \longrightarrow E}{\Delta, \nabla D, \Theta[\Gamma, A \cdot B, \Sigma], \chi \longrightarrow E}\ perm}\ cut \tag{145}$$

c. $\Theta = \Theta[C]$. This we transform to

$$\dfrac{\dfrac{\Gamma, A, B, \Sigma \longrightarrow C}{\Gamma, A \cdot B, \Sigma \longrightarrow C}\ \cdot_L \qquad \Delta, \nabla D, \Theta, \chi[C] \longrightarrow E}{\dfrac{\Delta, \Theta, \nabla D, \chi[\Gamma, A \cdot B, \Sigma] \longrightarrow E}{\Delta, \nabla D, \Theta, \chi[\Gamma, A \cdot B, \Sigma] \longrightarrow E}\ perm}\ cut \tag{146}$$

3. $\mathrm{fin}(\pi) = !_L$. In this case, the cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\ !_L \qquad \Theta[C] \longrightarrow B}{\Theta[\Gamma, !\Sigma, \Delta] \longrightarrow B}\ cut \tag{147}$$

we continue by analysing the possible endings of τ.

i. $\mathrm{fin}(\tau) = \backslash_L$. This cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\ !_L \qquad \dfrac{\chi \longrightarrow B \quad \Theta, C, \Psi \longrightarrow D}{\Theta, \chi, B\backslash C, \Psi \longrightarrow D}\ \backslash_L}{\Theta, \chi, B\backslash C, \Psi \longrightarrow D}\ cut \tag{148}$$

where the cut formula A is contained in Θ, χ or Ψ, giving us the following three cases.

a. $\Theta = \Theta[A]$. This is transformed to

$$\dfrac{\chi \longrightarrow B \qquad \dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\ !_L \qquad \Theta[A], C, \Psi \longrightarrow D}{\Theta[\Gamma, !\Sigma, \Delta], C, \Psi \longrightarrow D}\ cut}{\Theta[\Gamma, !\Sigma, \Delta], \chi, B\backslash C, \Psi \longrightarrow D}\ \backslash_L \tag{149}$$

b. $\chi = \chi[A]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \qquad \chi[A] \longrightarrow B}{\chi[\Gamma, !\Sigma, \Delta] \longrightarrow B}\,\mathit{cut} \qquad \Theta, C, \Psi \longrightarrow D}{\Theta, \chi[\Gamma, !\Sigma, \Delta], B\backslash C, \Psi \longrightarrow D}\,\backslash_L \tag{150}$$

c. $\Psi = \Psi[A]$. This is transformed to

$$\dfrac{\chi \longrightarrow B \qquad \dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \qquad \Theta, C, \Psi[A] \longrightarrow D}{\Theta, C, \Psi[\Gamma, !\Sigma, \Delta] \longrightarrow D}\,\mathit{cut}}{\Theta, \chi, B\backslash C, \Psi[\Gamma, !\Sigma, \Delta] \longrightarrow D}\,\backslash_L \tag{151}$$

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ii. $\mathrm{fin}(\tau) = \backslash_R$. In this case the cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \qquad \dfrac{B, \Theta[A] \longrightarrow C}{\Theta[A] \longrightarrow B\backslash C}\,\backslash_R}{\Theta[\Gamma, !\Sigma, \Delta] \longrightarrow B\backslash C}\,\mathit{cut} \tag{152}$$

which we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \qquad B, \Theta[A] \longrightarrow C}{B, \Theta[\Gamma, !\Sigma, \Delta] \longrightarrow C}\,\mathit{cut}}{\Theta[\Gamma, !\Sigma, \Delta] \longrightarrow B\backslash C}\,\backslash_R \tag{153}$$

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iii. $\mathrm{fin}(\tau) = \cdot_L$. In this case, the cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \qquad \dfrac{\Theta, B, C, \chi \longrightarrow D}{\Theta, B \cdot C, \chi \longrightarrow D}\,\cdot_L}{\Theta, B \cdot C, \chi \longrightarrow D}\,\mathit{cut} \tag{154}$$

where the cut formula A is contained in Θ or χ, giving us the following two cases

a. $\Theta = \Theta[A]$. This is transformed into

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \qquad \Theta[A], B, C, \chi \longrightarrow D}{\Theta[\Gamma, !\Sigma, \Delta], B, C, \chi \longrightarrow D}\,\mathit{cut}}{\Theta[\Gamma, !\Sigma, \Delta], B \cdot C, \chi \longrightarrow D}\,\cdot_L \tag{155}$$

b. $\chi = \chi[A]$. This is transformed into

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \qquad \Theta, B, C, \chi[A] \longrightarrow D}{\Theta, B, C, \chi[\Gamma, !\Sigma, \Delta] \longrightarrow D}\,\mathit{cut}}{\Theta, B \cdot C, \chi[\Gamma, !\Sigma, \Delta] \longrightarrow D}\,\cdot_L \tag{156}$$

iv. $\operatorname{fin}(\tau) = \cdot_R$. In this case the cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \quad \dfrac{\Theta \longrightarrow B \quad \chi \longrightarrow C}{\Theta, \chi \longrightarrow B \cdot C}\,\cdot_R}{\Theta, \chi \longrightarrow B \cdot C}\,cut \tag{157}$$

where the cut formula is contained in either Θ or χ, giving us the following two cases.

a. $\Theta = \Theta[A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \quad \Theta[A] \longrightarrow B}{\Theta[\Gamma, !\Sigma, \Delta] \longrightarrow B}\,cut \quad \chi \longrightarrow C}{\Theta[\Gamma, !\Sigma, \Delta], \chi \longrightarrow B \cdot C}\,\cdot_R \tag{158}$$

b. $\chi = \chi[A]$. This we transform to

$$\dfrac{\Theta \longrightarrow B \quad \dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \quad \Theta[A] \longrightarrow B}{\chi[\Gamma, !\Sigma, \Delta] \longrightarrow C}\,cut}{\Theta, \chi[\Gamma, !\Sigma, \Delta] \longrightarrow B \cdot C}\,\cdot_R \tag{159}$$

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v. $\operatorname{fin}(\tau) = !_L$. In this case the cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \quad \dfrac{\Theta, \chi, \chi, \ldots, \chi, \Psi \longrightarrow B}{\Theta, !\chi, \Psi \longrightarrow B}\,!_L}{\Theta, !\chi, \Psi \longrightarrow B}\,cut \tag{160}$$

where the cut formula A is contained in either Θ or Ψ, giving us the following two cases.

a. $\Theta = \Theta[A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \quad \Theta[A], \chi, \chi, \ldots, \chi, \Psi \longrightarrow B}{\Theta[\Gamma, !\Sigma, \Delta], \chi, \chi, \ldots, \chi, \Psi \longrightarrow B}\,!_L}{\Theta[\Gamma, !\Sigma, \Delta], !\chi, \Psi \longrightarrow B}\,!_L \tag{161}$$

b. $\Psi = \Psi[A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\,!_L \quad \Theta, \chi, \chi, \ldots, \chi, \Psi[A] \longrightarrow B}{\Theta, \chi, \chi, \ldots, \chi, \Psi[\Gamma, !\Sigma, \Delta] \longrightarrow B}\,!_L}{\Theta, !\chi, \Psi[\Gamma, !\Sigma, \Delta] \longrightarrow B}\,!_L \tag{162}$$

vi. $\mathrm{fin}(\tau) = !_R$. In this case no cut is possible.

vii. $\mathrm{fin}(\tau) = \nabla_L$. In this case the cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\ !_L \quad \dfrac{\Theta, B, \chi \longrightarrow C}{\Theta, \nabla B, \chi \longrightarrow C}\ \nabla_L}{\Theta, \nabla B, \chi \longrightarrow C}\ cut \tag{163}$$

where the cut formula A is contained in either Θ or chi, giving us the following two cases.

a. $\Theta = \Theta[A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\ !_L \quad \Theta[A], B, \chi \longrightarrow C}{\Theta[\Gamma, !\Sigma, \Delta], B, \chi \longrightarrow C}\ cut}{\Theta[\Gamma, !\Sigma, \Delta], \nabla B, \chi \longrightarrow C}\ \nabla_L \tag{164}$$

b. $\chi = \chi[A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\ !_L \quad \Theta, B, \chi[A] \longrightarrow C}{\Theta, B, \chi[\Gamma, !\Sigma, \Delta] \longrightarrow C}\ cut}{\Theta, \nabla B, \chi[\Gamma, !\Sigma, \Delta] \longrightarrow C}\ \nabla_L \tag{165}$$

viii. $\mathrm{fin}(\tau) = \nabla_R$. In this case no cut is possible.

ix. $\mathrm{fin}(\tau) = perm$. In this case the cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\ !_L \quad \dfrac{\Theta, \chi, \nabla B, \Psi \longrightarrow C}{\Theta, \nabla B, \chi, \Psi \longrightarrow C}\ perm}{\Theta, \nabla B, \chi, \Psi \longrightarrow C}\ cut \tag{166}$$

where the cut formula A is contained in either Θ, χ or Ψ, giving us the following three cases

a. $\Theta = \Theta[A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\ !_L \quad \Theta[A], \nabla B, \chi, \Psi \longrightarrow C}{\Theta[\Gamma, !\Sigma, \Delta], \chi, \nabla B, \Psi \longrightarrow C}\ cut}{\Theta[\Gamma, !\Sigma, \Delta], \nabla B, \chi, \Psi \longrightarrow C}\ perm \tag{167}$$

b. $\chi = \chi[A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\ !_L \quad \Theta, \nabla B, \chi[A], \Psi \longrightarrow C}{\Theta, \chi[\Gamma, !\Sigma, \Delta], \nabla B, \Psi \longrightarrow C}\ cut}{\Theta, \nabla B, \chi[\Gamma, !\Sigma, \Delta], \Psi \longrightarrow C}\ perm \tag{168}$$

c. $\Theta = \Theta[A]$. This we transform to

$$\dfrac{\dfrac{\Gamma, \Sigma, \Sigma, \ldots, \Sigma, \Delta \longrightarrow A}{\Gamma, !\Sigma, \Delta \longrightarrow A}\ !_L \qquad \Theta, \nabla B, \chi, \Psi[A] \longrightarrow C}{\dfrac{\Theta, \chi, \nabla B, \Psi[\Gamma, !\Sigma, \Delta] \longrightarrow C}{\Theta, \nabla B, \chi, \Psi[\Gamma, !\Sigma, \Delta] \longrightarrow C}\ perm}\ cut \tag{169}$$

4. $\text{fin}(\pi) = \nabla_L$. In this case the cut is

$$\dfrac{\dfrac{\Gamma, A, \Sigma}{\Gamma, \nabla A, \Sigma}\ \nabla_L \qquad \Theta[B] \longrightarrow C}{\Theta[\Gamma, \nabla A, \Sigma] \longrightarrow C}\ cut \tag{170}$$

we continue by analysing the possible final rules of τ.

i. $\text{fin}(\tau) = \backslash_L$. This cut looks like

$$\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\ \nabla_L \qquad \dfrac{\Delta \longrightarrow C \quad \Theta, D, \chi \longrightarrow E}{\Theta, \Delta, C\backslash D, \chi \longrightarrow E}\ \backslash_L}{\Theta, \Delta, C\backslash D, \chi \longrightarrow E}\ cut \tag{171}$$

where the cut formula B is contained in either Θ, Δ or χ, giving us the following three cases.

a. $\Theta = \Theta[A]$. This we transform to

$$\dfrac{\Delta \longrightarrow C \qquad \dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\ \nabla_L \quad \Theta[B], D, \chi \longrightarrow E}{\Theta[\Gamma, \nabla A, \Sigma], D, \chi \longrightarrow E}\ cut}{\Theta[\Gamma, \nabla A, \Sigma], \Delta, C\backslash D, \chi \longrightarrow E}\ \backslash_L \tag{172}$$

b. $\Delta = \Delta[A]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\ \nabla_L \quad \Delta[B] \longrightarrow C}{\Delta[\Gamma, \nabla A, \Sigma] \longrightarrow C}\ cut \qquad \Theta, D, \chi \longrightarrow E}{\Theta, \Delta[\Gamma, \nabla A, \Sigma], C\backslash D, \chi \longrightarrow E}\ \backslash_L \tag{173}$$

c. $\chi = \chi[A]$. This we transform to

$$\dfrac{\Delta \longrightarrow C \qquad \dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\ \nabla_L \quad \Theta, D, \chi[B] \longrightarrow E}{\Theta, D, \chi[\Gamma, \nabla A, \Sigma] \longrightarrow E}\ cut}{\Theta, \Delta, C\backslash D, \chi[\Gamma, \nabla A, \Sigma] \longrightarrow E}\ \backslash_L \tag{174}$$

ii. $\mathrm{fin}(\tau) = \backslash_R$. This cut looks like

$$\frac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\,\nabla_L \quad \dfrac{C, \Delta[B] \longrightarrow D}{\Delta[B] \longrightarrow C\backslash D}\,\backslash_R}{\Delta[\Gamma, \nabla A, \Sigma] \longrightarrow C\backslash D}\,cut \tag{175}$$

which we transform into

$$\frac{\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\,\nabla_L \quad C, \Delta[B] \longrightarrow D}{C, \Delta[\Gamma, \nabla A, \Sigma] \longrightarrow D}\,cut}{\Delta[\Gamma, \nabla A, \Sigma] \longrightarrow C\backslash D}\,\backslash_R \tag{176}$$

iii. $\mathrm{fin}(\tau) = \cdot_L$. This cut looks like

$$\frac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\,\nabla_L \quad \dfrac{\Delta, C, D, \Theta \longrightarrow E}{\Delta, C \cdot D, \Theta \longrightarrow E}\,cdot_L}{\Delta, C \cdot D, \Theta \longrightarrow E}\,cut \tag{177}$$

where the cut formula is contained in either Δ or Θ, giving us the following two cases

a. $\Delta = \Delta[B]$. This we transform to

$$\frac{\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\,\nabla_L \quad \Delta[B], C, D, \Theta \longrightarrow E}{\Delta[\Gamma, \nabla A, \Sigma], C, D, \Theta \longrightarrow E}\,cut}{\Delta[\Gamma, \nabla A, \Sigma], C \cdot D, \Theta \longrightarrow E}\,\cdot_L \tag{178}$$

b. $\Theta = \Theta[B]$. This we transform to

$$\frac{\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\,\nabla_L \quad \Delta, C, D, \Theta[B] \longrightarrow E}{\Delta, C, D, \Theta[\Gamma, \nabla A, \Sigma] \longrightarrow E}\,cut}{\Delta, C \cdot D, \Theta[\Gamma, \nabla A, \Sigma] \longrightarrow E}\,\cdot_L \tag{179}$$

iv. $\mathrm{fin}(\tau) = \cdot_R$. This cut looks like

$$\frac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\,\nabla_L \quad \dfrac{\Delta \longrightarrow C \quad \Theta \longrightarrow D}{\Delta, \Theta \longrightarrow C \cdot D}\,\cdot_R}{\Delta, \Theta \longrightarrow C \cdot D}\,cut \tag{180}$$

where the cut formula is contained in either Δ or Θ, giving us the following two cases

a. $\Delta = \Delta[B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\ \nabla_L \quad \Delta[B] \longrightarrow C}{\Delta[\Gamma, \nabla A, \Sigma] \longrightarrow C}\ cut \quad \Theta \longrightarrow D}{\Delta[\Gamma, \nabla A, \Sigma], \Theta \longrightarrow C \cdot D}\ \cdot_R \tag{181}$$

b. $\Theta = \Theta[B]$. This we transform to

$$\dfrac{\Delta \longrightarrow C \quad \dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\ \nabla_L \quad \Theta[B] \longrightarrow D}{\Theta[\Gamma, \nabla A, \Sigma] \longrightarrow D}\ cut}{\Delta, \Theta[\Gamma, \nabla A, \Sigma] \longrightarrow C \cdot D}\ \cdot_R \tag{182}$$

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v. $\text{fin}(\tau) = !_L$. this cut looks like

$$\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\ \nabla_L \quad \dfrac{\Delta, \Theta, \Theta, \ldots, \Theta, \chi \longrightarrow C}{\Delta, !\Theta, \chi \longrightarrow C}\ !_L}{\Delta, !\Theta, \chi \longrightarrow C}\ cut \tag{183}$$

where the cut formula B is contained in either Δ or χ, giving us the following two cases.

a. $\Delta = \Delta[B]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\ \nabla_L \quad \Delta[B], \Theta, \Theta, \ldots, \Theta, \chi \longrightarrow C}{\Delta[\Gamma, \nabla A, \Sigma], \Theta, \Theta, \ldots, \Theta, \chi \longrightarrow C}\ cut}{\Delta[\Gamma, \nabla A, \Sigma], !\Theta, \chi \longrightarrow C}\ !_L \tag{184}$$

b. $\chi = \chi[B]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\ \nabla_L \quad \Delta, \Theta, \Theta, \ldots, \Theta, \chi[B] \longrightarrow C}{\Delta, \Theta, \Theta, \ldots, \Theta, \chi[\Gamma, \nabla A, \Sigma] \longrightarrow C}\ cut}{\Delta, !\Theta, \chi[\Gamma, \nabla A, \Sigma] \longrightarrow C}\ !_L \tag{185}$$

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vi. $\text{fin}(\tau) = !_R$. In this case no cut is possible.

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vii. $\text{fin}(\tau) = \nabla_L$. This cut looks like

$$\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow C}{\Gamma, \nabla A, \Sigma \longrightarrow C}\ \nabla_L \quad \dfrac{\Delta, C, \Theta \longrightarrow D}{\Delta, \nabla C, \Theta \longrightarrow D}\ \nabla_L}{\Delta, \nabla C, \Theta \longrightarrow D}\ cut \tag{186}$$

where the cut formula B is contained in either Δ or Θ, giving us the following two cases.

a. $\Delta = \Delta[B]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow C}{\Gamma, \nabla A, \Sigma \longrightarrow C}\,\nabla_L \qquad \Delta[B], \nabla C, \Theta \longrightarrow D}{\Delta[\Gamma, A, \Sigma], \nabla C, \Theta \longrightarrow D}\,\mathit{cut}}{\Delta[\Gamma, \nabla A, \Sigma], \nabla C, \Theta \longrightarrow D}\,\nabla_L \tag{187}$$

b. $\Theta = \Theta[B]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow C}{\Gamma, \nabla A, \Sigma \longrightarrow C}\,\nabla_L \qquad \Delta, \nabla C, \Theta[B] \longrightarrow D}{\Delta, \nabla C, \Theta[\Gamma, \nabla A, \Sigma] \longrightarrow D}\,\mathit{cut}}{\Delta, \nabla C, \Theta[\Gamma, \nabla A, \Sigma] \longrightarrow D}\,\nabla_L \tag{188}$$

viii. $\operatorname{fin}(\tau) = \nabla_R$. In this case no cut is possible.

ix. $\operatorname{fin}(\tau) = \mathit{perm}$. This cut looks like

$$\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\,\nabla_L \qquad \dfrac{\Delta, \Theta, \nabla C, \chi \longrightarrow D}{\Delta, \nabla C, \Theta, \chi \longrightarrow D}\,\mathit{perm}}{\Delta, \nabla C, \Theta, \chi \longrightarrow D}\,\mathit{cut} \tag{189}$$

where the cut formula B is contained in either Δ, Θ or χ, giving us the following three cases.

a. $\Delta = \Delta[B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\,\nabla_L \qquad \Delta[B], \Theta, \nabla C, \chi \longrightarrow D}{\Delta[\Gamma, \nabla A, \Sigma], \Theta, \nabla C, \chi \longrightarrow D}\,\mathit{cut}}{\Delta[\Gamma, \nabla A, \Sigma], \nabla C, \Theta, \chi \longrightarrow D}\,\mathit{perm} \tag{190}$$

b. $\Theta = \Theta[B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\,\nabla_L \qquad \Delta, \Theta[B], \nabla C, \chi \longrightarrow D}{\Delta, \Theta[\Gamma, \nabla A, \Sigma], \nabla C, \chi \longrightarrow D}\,\mathit{cut}}{\Delta, \nabla C, \Theta[\Gamma, \nabla A, \Sigma], \chi \longrightarrow D}\,\mathit{perm} \tag{191}$$

c. $\chi = \chi[B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, A, \Sigma \longrightarrow B}{\Gamma, \nabla A, \Sigma \longrightarrow B}\,\nabla_L \qquad \Delta, \Theta, \nabla C, \chi[B] \longrightarrow D}{\Delta, \Theta, \nabla C, \chi[\Gamma, \nabla A, \Sigma] \longrightarrow D}\,\mathit{cut}}{\Delta, \nabla C, \Theta, \chi[\Gamma, \nabla A, \Sigma] \longrightarrow D}\,\mathit{perm} \tag{192}$$

Case III

$\mathrm{fin}(\pi) = perm$. In this case the cut is of the form

$$\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\ \mathit{perm} \qquad \Theta[B] \longrightarrow C}{\Theta[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow C}\ \mathit{cut} \tag{193}$$

we continue by analysing the possible endings of τ.

i. $\mathrm{fin}(\tau) = \backslash_L$. This cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\ \mathit{perm} \qquad \dfrac{\chi \longrightarrow C \quad \Theta, D, \Psi \longrightarrow E}{\Theta, \chi, C\backslash D, \Psi \longrightarrow E}\ \backslash_L}{\Theta, \chi, C\backslash D, \Psi \longrightarrow E}\ \mathit{cut} \tag{194}$$

where the cut formula B is contained in either Θ, χ or Ψ, giving us the following three cases

a. $\Theta = \Theta[B]$. This we transform to

$$\dfrac{\chi \longrightarrow C \qquad \dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\ \mathit{perm} \quad \Theta[B], D, \Psi \longrightarrow E}{\Theta[\Gamma, \nabla A, \Sigma, \Delta], D, \Psi \longrightarrow E}\ \mathit{cut}}{\Theta[\Gamma, \nabla A, \Sigma, \Delta], \chi, C\backslash D, \Psi \longrightarrow E}\ \backslash_L \tag{195}$$

b. $\chi = \chi[B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\ \mathit{perm} \quad \chi[B] \longrightarrow C}{\chi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow C}\ \mathit{cut} \qquad \Theta, D, \Psi \longrightarrow E}{\Theta, \chi[\Gamma, \nabla A, \Sigma, \Delta], C\backslash D, \Psi \longrightarrow E}\ \backslash_L \tag{196}$$

c. $\Psi = \Psi[B]$. This we transform to

$$\dfrac{\chi \longrightarrow C \qquad \dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\ \mathit{perm} \quad \Theta, D, \Psi[B] \longrightarrow E}{\Theta, D, \Psi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow E}\ \mathit{cut}}{\Theta, \chi, C\backslash D, \Psi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow E}\ \backslash_L \tag{197}$$

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ii. $\mathrm{fin}(\tau) = \backslash_R$. This cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\ \mathit{perm} \qquad \dfrac{C, \Theta[B] \longrightarrow D}{\Theta[B] \longrightarrow C\backslash D}\ \backslash_R}{\Theta[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow C\backslash D}\ \mathit{cut} \tag{198}$$

which we transform into

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\ perm \quad C, \Theta[B] \longrightarrow D}{C, \Theta[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow D}\ cut}{\Theta[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow C\backslash D}\ \backslash_R \tag{199}$$

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iii. $\text{fin}(\tau) = \cdot_L$. This cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\ perm \quad \dfrac{\Theta, C, D, \chi \longrightarrow E}{\Theta, C \cdot D, \chi \longrightarrow E}\ \cdot_L}{\Theta, C \cdot D, \chi \longrightarrow E}\ cut \tag{200}$$

where the cut formula B is contained in either Θ or χ, giving us the following two cases.

a. $\Theta = \Theta[B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\ perm \quad \Theta[B], C, D, \chi \longrightarrow E}{\Theta[\Gamma, \nabla A, \Sigma, \Delta], C, D, \chi \longrightarrow E}\ cut}{\Theta[\Gamma, \nabla A, \Sigma, \Delta], C \cdot D, \chi \longrightarrow E}\ \cdot_L \tag{201}$$

b. $\chi = \chi[B]$. This we transform to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\ perm \quad \Theta, C, D, \chi[B] \longrightarrow E}{\Theta, C, D, \chi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow E}\ cut}{\Theta, C \cdot D, \chi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow E}\ \cdot_L \tag{202}$$

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iv. $\text{fin}(\tau) = \cdot_R$. This cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\ perm \quad \dfrac{\Theta \longrightarrow C \quad \chi \longrightarrow D}{\Theta, \chi \longrightarrow C \cdot D}\ \cdot_R}{\Theta, \chi \longrightarrow C \cdot D}\ cut \tag{203}$$

where the cut formula B is contained in either Θ or χ, giving us the following two cases.

a. $\Theta = \Theta[B]$.

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\ perm \quad \Theta[B] \longrightarrow C}{\Theta[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow C}\ cut \quad \chi \longrightarrow D}{\Theta[\Gamma, \nabla A, \Sigma, \Delta], \chi \longrightarrow C \cdot D}\ \cdot_R \tag{204}$$

b. $\Theta = \Theta[B]$.

$$\dfrac{\Theta \longrightarrow C \qquad \dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\,\mathit{perm} \qquad \chi[B] \longrightarrow D}{\chi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow D}\,\mathit{cut}}{\Theta, \chi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow C \cdot D}\,\cdot_R \tag{205}$$

---

v. $\mathrm{fin}(\tau) = !_L$. This cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\,\mathit{perm} \qquad \dfrac{\Theta, \chi, \chi, \ldots, \chi, \Psi \longrightarrow C}{\Theta, !\chi, \Psi \longrightarrow C}\,!_L}{\Theta, !\chi, \Psi \longrightarrow C}\,\mathit{cut} \tag{206}$$

where the cut formula is contained in either Θ or Ψ, giving us the following two cases.

a. $\Theta = \Theta[B]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\,\mathit{perm} \qquad \Theta[B], \chi, \chi, \ldots, \chi, \Psi \longrightarrow C}{\Theta[\Gamma, \nabla A, \Sigma, \Delta], \chi, \chi, \ldots, \chi, \Psi \longrightarrow C}\,\mathit{cut}}{\Theta[\Gamma, \nabla A, \Sigma, \Delta], !\chi, \Psi \longrightarrow C}\,!_L \tag{207}$$

b. $\chi = \chi[B]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\,\mathit{perm} \qquad \Theta, \chi, \chi, \ldots, \chi, \Psi[B] \longrightarrow C}{\Theta, \chi, \chi, \ldots, \chi, \Psi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow C}\,\mathit{cut}}{\Theta, !\chi, \Psi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow C}\,!_L \tag{208}$$

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vi. $\mathrm{fin}(\tau) = !_R$. In this case no cut is possible.

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vii. $\mathrm{fin}(\tau) = \nabla_L$. This cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\,\mathit{perm} \qquad \dfrac{\Theta, C, \chi \longrightarrow D}{\Theta, \nabla C, \chi \longrightarrow D}\,\nabla_L}{\Theta, \nabla C, \chi \longrightarrow D}\,\mathit{cut} \tag{209}$$

where the cut formula B is contained in either Θ or χ, giving us the following two cases.

a. $\Theta = \Theta[B]$. This is transformed to

$$\dfrac{\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\,\mathit{perm} \qquad \Theta[B], C, \chi \longrightarrow D}{\Theta[\Gamma, \nabla A, \Sigma, \Delta], C, \chi \longrightarrow D}\,\mathit{cut}}{\Theta[\Gamma, \nabla A, \Sigma, \Delta], \nabla C, \chi \longrightarrow D}\,\nabla_L \tag{210}$$

b. $\chi = \chi[B]$. This is transformed to

$$\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\, perm \quad \Theta, C, \chi[B] \longrightarrow D}{\dfrac{\Theta, C, \chi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow D}{\Theta, \nabla C, \chi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow D}\, \nabla_L}\, cut \tag{211}$$

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viii. $\mathrm{fin}(\tau) = \nabla_R$. In this case no cut is possible.

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ix. $\mathrm{fin}(\tau) = perm$. This cut looks like

$$\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\, perm \quad \dfrac{\Theta, \chi, \nabla C, \Psi \longrightarrow D}{\Theta, \nabla C, \chi, \Psi \longrightarrow D}\, perm}{\Theta, \nabla C, \chi, \Psi \longrightarrow D}\, cut \tag{212}$$

where the cut formula B is contained in either Θ, χ or Ψ, giving us the following three cases

a. $\Theta = \Theta[B]$. This is transformed to

$$\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\, perm \quad \Theta[B], \chi, \nabla C, \Psi \longrightarrow D}{\dfrac{\Theta[\Gamma, \nabla A, \Sigma, \Delta], \chi, \nabla C, \Psi \longrightarrow D}{\Theta[\Gamma, \nabla A, \Sigma, \Delta], \nabla C, \chi, \Psi \longrightarrow D}\, perm}\, cut \tag{213}$$

b. $\chi = \chi[B]$. This is transformed to

$$\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\, perm \quad \Theta, \chi[B], \nabla C, \Psi \longrightarrow D}{\dfrac{\Theta, \chi[\Gamma, \nabla A, \Sigma, \Delta], \nabla C, \Psi \longrightarrow D}{\Theta, \nabla C, \chi[\Gamma, \nabla A, \Sigma, \Delta], \Psi \longrightarrow D}\, perm}\, cut \tag{214}$$

c. $\Psi = \Psi[B]$. This is transformed to

$$\dfrac{\dfrac{\Gamma, \Sigma, \nabla A, \Delta \longrightarrow B}{\Gamma, \nabla A, \Sigma, \Delta \longrightarrow B}\, perm \quad \Theta, \chi, \nabla C, \Psi[B] \longrightarrow D}{\dfrac{\Theta, \chi, \nabla C, \Psi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow D}{\Theta, \nabla C, \chi, \Psi[\Gamma, \nabla A, \Sigma, \Delta] \longrightarrow D}\, perm}\, cut \tag{215}$$

□